Fully microwave-tunable universal gates in superconducting qubits with linear couplings and fixed transition frequencies

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A register of quantum bits with fixed transition frequencies and weakly coupled to one another through simple linear circuit elements is an experimentally minimal architecture for a small-scale superconducting quantum information processor. Presently, the known schemes for implementing two-qubit gates in this system require microwave signals having amplitudes and frequencies precisely tuned to meet a resonance condition, leaving only the signal phases as free experimentally adjustable parameters. Here, we report a minimal and robust microwave scheme to generate fast, tunable universal two-qubit gates: simply irradiate one qubit (the "control") at the transition frequency of another (the "target"). The effective coupling between them is then switched on by tuning only the frequency of this single drive tone; the drive amplitude adjusts the effective coupling strength; and the drive phase selects the particular two-qubit gate implemented. This cross-resonance effect turns on linearly with the ratio of the drive amplitude Ω to the qubit-qubit detuning Δ, as compared with earlier proposals that turn on as $(\Omega/\Delta)^4$.

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I. INTRODUCTION

Superconducting quantum circuits are a promising candidate technology for a quantum information processor, with several important experimental milestones having recently been achieved.[1–7] Entanglement-generating two-qubit gates have been implemented via coupling through an on-chip cavity mode by Sillanpää *et al.*[1] and by Majer *et al.*;[2] a microwave-controlled tunable nonlinear subcircuit by Niskanen *et al.*;[3] and via coupling through fixed linear subcircuits by Plantenberg *et al.*[4] More recently, rudimentary algorithms have been demonstrated in two qubits coupled through energy states outside the computational subspace;[5] a violation of Bell's inequalities has been measured[6] and the Mollow sidebands[8] of a qubit under strong drive have been observed.[9] In most of these strategies, though the systems are biased with dc signals, two-qubit gates are generated by applying microwave signals of appropriate frequency, amplitude, and phase. In the case of cavity coupling,[2] the frequency and power are selected to ac Stark shift the two qubits into resonance. These results are an important step forward; among other things, they demonstrate that a single microwave control signal and fixed linear couplings can be used to perform two-qubit gates. Yet despite its advantages, the ac Stark-based gates offer little hope for scaling beyond registers of a few qubits. The experiments of Niskanen *et al.*[10,11] used a nonlinear coupling subcircuit driven in the linear regime at the difference frequency Δ of the two qubits. This parametric pumping scheme has the important advantage of tunable two-qubit gate speeds, as the strength of the effective qubit-qubit interaction increases linearly with the drive amplitude Ω. Such tunability is especially important for applications to flux qubits and other designs in which the obtained qubit transition frequencies tend to have a broad distribution.

In this paper we describe a technique for entangling superconducting qubits that synthesizes the three desirable properties of fixed linear couplings, exclusively microwave control signals, and tunable effective interaction strengths. Using only fixed linear couplings requires no additional control or bias lines beyond those used for the manipulation of individual qubits. This configuration minimizes both experimental complexity and the unwanted coupling of circuit modes to degrees of freedom in the electromagnetic environment—two important practical properties for controlling decoherence and scaling to many-qubit systems. In addition, coupling the qubits through simple on-chip inductances or capacitances simplifies and makes more reliable standard circuit fabrication. Critically, relying on purely microwave signals for implementing nonlocal gate operations frees up the available dc controls for other uses, such as optimizing qubit coherence[12] or one-qubit gate fidelity; tuning the qubit-readout coupling;[13] or for simplifying and parallelizing the one-qubit operations. Last, tuning the effective interaction strength allows us to entangle pairs of qubits having large Δ—a tremendous advantage relative to earlier fixed-coupling proposals which required comparatively small interqubit detunings.[14–16]

II. DERIVATION OF THE CROSS-RESONANCE EFFECT

We employ in this proposal the FLICFORQ—*F*ixed *LI*near *C*ouplings between *F*ixed *O*ff-*R*esonant *Q*ubits—architecture studied in earlier work on microwave-controlled two-qubit gates.[14–17] However, we do not place any *a priori* restrictions on the viable circuit parameters beyond the weak-coupling constraint that ensures the qubits remain effectively decoupled in the absence of control signals.[18]

For a quantum register of this style, we present the following minimal two-qubit gate scheme: simply irradiate one of them at the transition frequency of the other. In the presence of this cross-resonant microwave drive, an effective coupling emerges between the two qubits whose strength increases linearly with the ratio (Ω/Δ).

In what follows, we derive the effect by moving to a special rotating frame that precesses with the driven dynamics of each qubit. In that frame we obtain a purely nonlocal Hamiltonian, i.e., one containing only coupling terms. We identify static terms in this Hamiltonian that dominate the system evolution when cross-resonant irradiation is applied. We then take into account the important practical issue of microwave signal cross-talk and show how it modifies the cross-resonant condition. We give an intuitive explanation of the effect in the dressed state picture[19] for both the zero and finite cross-talk cases. Last, we discuss the implications of this entangling mode for the fidelity of standard experimental schemes for demonstrating one-qubit control and implementing one-qubit gates.

For simplicity, our derivation of the cross-resonant coupling effect is done under the strict two-level approximation and in the absence of noise and decoherence. The two-qubit Hamiltonian with linear off-diagonal coupling and microwave irradiation is ($\hbar=1$ throughout),

$$\mathcal{H}=\frac{1}{2}\omega_1\sigma_1^z+\Omega_1\cos(\omega_1^{\mathrm{rf}}t+\phi_1)\sigma_1^x+\frac{1}{2}\omega_2\sigma_2^z+\Omega_2\cos(\omega_2^{\mathrm{rf}}t+\phi_2)\sigma_2^x+\frac{1}{2}\omega_{xx}\sigma_1^x\sigma_2^x, \quad (1)$$

where $\omega_j/2\pi$ is the transition frequency of qubit j; Ω_j and $\omega_j^{\mathrm{rf}}/2\pi$ are, respectively, the amplitude and frequency of the microwave signal applied to qubit j; and $\omega_{xx}/2\pi$ is the coupling energy. To ensure the qubits remain decoupled in the absence of control signals, we impose the weak-coupling constraint, $\omega_{xx}\ll|\Delta|\equiv|\omega_2-\omega_1|$.

To derive the cross-resonant coupling Hamiltonian we first take $\mathcal{H}$ through a series of unitary transformations which transfer the dynamics of the system from the local one-qubit terms to the nonlocal coupling term, ultimately arriving at a purely nonlocal[21] Hamiltonian.[22,23] Though our protocol requires only that one qubit be irradiated, we leave the microwave control parameters $\{\Omega_{1,2},\omega_{1,2}^{\mathrm{rf}},\phi_{1,2}\}$ general for the time being, as this will facilitate the cross-talk analysis to follow. Later, we will impose the cross-resonant conditions on the resulting Hamiltonian and see that static terms emerge as a result.

First, we take the laboratory-frame Hamiltonian (1) to a reference frame that rotates about the σ_1^z and σ_2^z axes with the drive signals of frequency ω_1^{rf} and ω_2^{rf} (respectively) with the unitary transformation,

$$\mathcal{U}_a=\exp[-it(\omega_1^{\mathrm{rf}}\sigma_1^z+\omega_2^{\mathrm{rf}}\sigma_2^z)/2]. \quad (2)$$

We make a rotating wave approximation on each irradiated qubit by neglecting terms oscillating at $2\omega_j^{\mathrm{rf}}$, then make a time-independent rotation about $\sigma_{1,2}^z$ to null the microwave signal phases with

$$\mathcal{U}_b=\exp[-i(\phi_1\sigma_1^z+\phi_2\sigma_2^z)/2], \quad (3)$$

which brings the residual static terms into the original xz plane. According to Ref. 24,

$$\mathcal{H}'=\mathcal{U}^{-1}\mathcal{H}\mathcal{U}-i\mathcal{U}^{-1}\,\partial\mathcal{U}/\partial t, \quad (4)$$

the transformed Hamiltonian becomes, after these steps,

$$2\mathcal{H}_{\mathrm{DF}}=\delta_1\sigma_1^z+\Omega_1\sigma_1^x+\delta_2\sigma_2^z+\Omega_2\sigma_2^x+\omega_{xx}[\sigma_1^x\cos(\phi_1+\omega_1^{\mathrm{rf}}t)-\sigma_1^y\sin(\phi_1+\omega_1^{\mathrm{rf}}t)][\sigma_2^x\cos(\phi_2+\omega_2^{\mathrm{rf}}t)-\sigma_2^y\sin(\phi_2+\omega_2^{\mathrm{rf}}t)], \quad (5)$$

where $\delta_j=\omega_j-\omega_j^{\mathrm{rf}}$ denotes the qubit-field detuning. The subscript DF denotes the doubly rotating reference frame to which we return later.

In this frame the qubits are subject only to static local fields of strengths $\eta_j\equiv(\delta_j^2+\Omega_j^2)^{1/2}$ while the once static coupling has acquired the time dependence of the drive fields. Next, we tilt the one-qubit static fields about $\sigma_{1,2}^y$ by angles,

$$\xi_j\equiv\tan^{-1}(\delta_j/\Omega_j) \quad (6)$$

with the time-independent unitary transformation,

$$\mathcal{U}_c=\exp[-i(\xi_1\sigma_1^y+\xi_2\sigma_2^y)/2]. \quad (7)$$

This aligns the residual local fields η_j along the σ_j^x axes. Last, we apply a second set of time-dependent transformations,

$$\mathcal{U}_d=\exp[-it(\eta_1\sigma_1^x+\eta_2\sigma_2^x)/2], \quad (8)$$

which null the remaining local terms.

Through this series of transformations, comprising two time-independent and two time-dependent rotations on each qubit, $\mathcal{H}$ is transferred from the laboratory frame to the particular quadruply rotating reference frame where the full system dynamics are contained in a purely nonlocal Hamiltonian. We call this expression the *quad frame Hamiltonian*, $\mathcal{H}_{\mathrm{QF}}$, and note that it is yet an exact description of the original system—up to the standard rotating wave approximation we have made on each driven qubit. We have

$$\mathcal{H}_{\mathrm{QF}}=\frac{\omega_{xx}}{2}\{\cos(\omega_1^{\mathrm{rf}}t+\phi_1)[\sigma_1^x\cos\xi_1-(\sigma_1^z\cos\eta_1t+\sigma_1^y\sin\eta_1t)\sin\xi_1]-\sin(\omega_1^{\mathrm{rf}}t+\phi_1)(\sigma_1^y\cos\eta_1t-\sigma_1^z\sin\eta_1t)\}\times\{\cos(\omega_2^{\mathrm{rf}}t+\phi_2)[\sigma_2^x\cos\xi_2-(\sigma_2^z\cos\eta_2t+\sigma_2^y\sin\eta_2t)\sin\xi_2]-\sin(\omega_2^{\mathrm{rf}}t+\phi_2)\times(\sigma_2^y\cos\eta_2t-\sigma_2^z\sin\eta_2t)\}. \quad (9)$$

Without further constraints on system parameters, $\mathcal{H}_{\mathrm{QF}}$ contains only rapidly oscillating terms, indicating that the fixed weak off-diagonal coupling enters only to second order in ω_{xx}/Δ. The selection of appropriate drive parameters, however, causes certain terms in $\mathcal{H}_{\mathrm{QF}}$ to become time independent. The original FLICFORQ proposal[14] focused on the case where $\omega_1^{\mathrm{rf}}=\omega_1$ and $\omega_2^{\mathrm{rf}}=\omega_2$, which led to time-independent terms when the drive amplitudes satisfied $\Omega_1+\Omega_2=\Delta$. By considering also off-resonant driving, this was extended[15] to the more general condition $\omega_1^{\mathrm{rf}}-\omega_2^{\mathrm{rf}}=\eta_1+\eta_2$.

Here we consider the case where a cross-resonant tone is applied and show that this leads to a static term in $\mathcal{H}_{\mathrm{QF}}$ which is switched on not by the signal amplitude but the signal frequency. This offers a significant practical advantage, as microwave frequencies may be controlled with greater precision than amplitudes in most experiments. Further, we shall show that the strength of the effective interac-

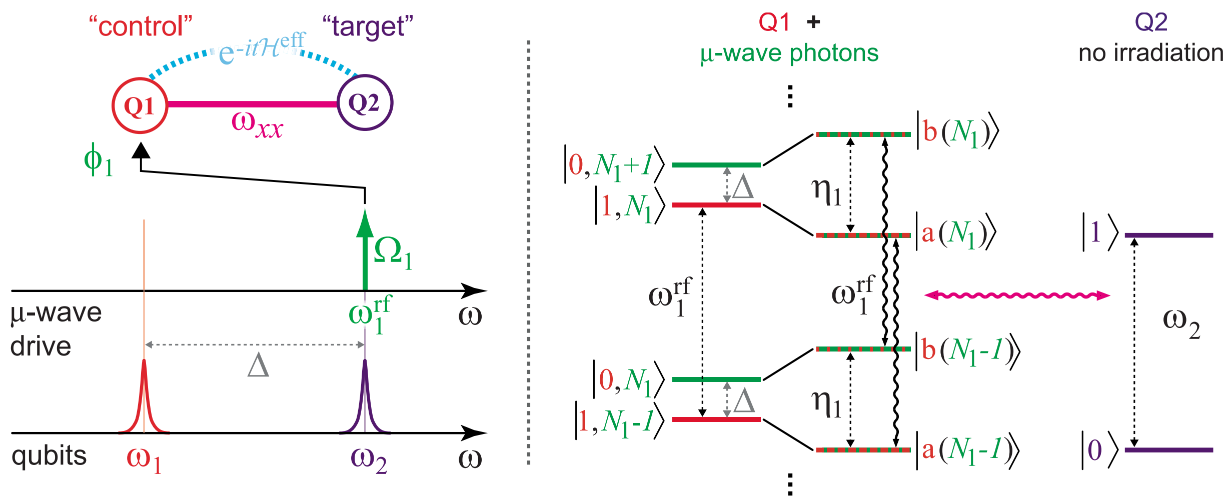

FIG. 1. (Color online) Left: FLICFORQ-style qubits (circles) have fixed transition frequencies ω_j and fixed linear off-diagonal coupling ω_{xx} (magenta bar). One- and two-qubit gates are performed by applying exclusively microwave control signals. Circuit parameters are selected to satisfy the weak-coupling condition $\omega_{xx} \ll |\Delta| \equiv |\omega_1 - \omega_2|$, allowing the interaction to be neglected in the absence of irradiation up to an error oscillation of frequency $(\omega_{xx}^2 + \Delta^2)^{1/2}$ and amplitude ω_{xx}/Δ. Cross resonance: a universal two-qubit gate can be implemented in this system by simply applying a microwave drive (green arrow) to the "control" qubit (Q1, red) at the transition frequency of the "target" (Q2, purple). This procedure generates the effective Hamiltonian $\mathcal{H}^{\text{eff}} = \frac{\Omega_1 \omega_{xx}}{4\Delta}(\sigma_1^z \sigma_2^x \cos \phi_1 + \sigma_1^z \sigma_2^y \sin \phi_1)$ and with $\phi_1 = 0$ the universal gate $[ZX]^{1/2}$ is realized in time $t = \pi\Delta/(\Omega_1 \omega_{xx})$. Right: cross resonance in the dressed state picture. When Q1 is irradiated at $\omega_1^{\text{rf}} = \omega_2$ the Q1+μ-wave photons system may undergo spontaneous transitions (Ref. 20) at ω_2. The interaction is switched on by tuning only the frequency of a single tone applied to one of the qubits and the resulting effective interaction strength turns on linearly with Ω_1/Δ.

tion grows linearly in the weak driving limit.

We now assume Q1 is irradiated at the frequency of Q2 while Q2 is left alone, as depicted in Fig. 1. In the idealized case of zero microwave cross-talk (Q2 senses no irradiation) we impose on $\mathcal{H}_{\text{QF}}$ the constraints,

$$\{\omega_1^{\text{rf}} \to \omega_2, \quad \xi_2 \to 0, \quad \eta_2 \to 0, \quad \phi_2 \to 0\}. \tag{10}$$

A static term then develops in $\mathcal{H}_{\text{QF}}$ that dominates the oscillating terms, whose effects are rapidly averaged out, leading to the effective Hamiltonian,

$$\mathcal{H}_{\text{QF}}^{\text{eff}} = \frac{1}{2}\omega_{xx}^{\text{eff}}(\sigma_1^x \sigma_2^x \cos \phi_1 + \sigma_1^x \sigma_2^y \sin \phi_1), \tag{11}$$

where

$$\omega_{xx}^{\text{eff}} = \frac{\omega_{xx}}{2}\cos \xi_1 = \frac{\omega_{xx}}{2\sqrt{1 + (\Delta/\Omega_1)^2}}, \tag{12}$$

indicating that for fixed circuit parameters ω_{xx}, ω_1, and ω_2, the interaction strength increases with the drive amplitude Ω_1.

Most practical measurements of qubit observables are performed in the doubly rotating frame. We transform this result back to that frame by inverting (in this order) the time-dependent rotations by angles $\eta_{1,2}t$ and the time-independent rotations by angles $\xi_{1,2}$ on each qubit. There we have, in terms of the interqubit detuning Δ and drive strength Ω_1,

$$\mathcal{H}_{\text{DF}}^{\text{eff}} = \frac{\omega_{xx}}{4}\frac{1}{1 + \Delta^2/\Omega_1^2} \times \left(\cos \phi_1 \sigma_1^x \sigma_2^x + \sin \phi_1 \sigma_1^x \sigma_2^y + \frac{\Delta}{\Omega_1}\cos \phi_1 \sigma_1^z \sigma_2^x + \frac{\Delta}{\Omega_1}\sin \phi_1 \sigma_1^z \sigma_2^y\right). \tag{13}$$

The cross-resonance effect thus turns on linearly in the ratio Ω_1/Δ. This is in sharp contrast with off-resonant FLICFORQ where the effect turns on as $(\Omega/\Delta)^4$.[17] For most practical implementations of this protocol the driving will satisfy $\Omega_1 \ll \Delta$, and only the $\sigma_1^z \sigma_2^x$ and $\sigma_1^z \sigma_2^y$ terms are important. We then have

$$\mathcal{H}_{\text{DF}}^{\text{eff}} \approx \frac{\Omega_1 \omega_{xx}}{4\Delta}(\cos \phi_1 \sigma_1^z \sigma_2^x + \sin \phi_1 \sigma_1^z \sigma_2^y). \tag{14}$$

A cross-resonant microwave pulse with $\phi_1 = 0$ thus implements the unitary transformation $\exp(-i\beta\pi\sigma_1^z \sigma_2^x/2) \equiv [ZX]^{\beta}$, which along with one-qubit unitaries is universal for quantum computation. By choosing the pulse parameters Ω_1 and t such that $\beta = \Omega_1 \omega_{xx} t/(2\pi\Delta) = 1/2$, one obtains the Clifford group generator $[ZX]^{1/2}$, which is related to the canonical two-qubit gate CNOT by only one additional local $\pi/2$ rotation of each qubit,[25,26]

$$\text{CNOT} = [ZI]^{-1/2}[ZX]^{1/2}[IX]^{-1/2}.$$

Comparing with resonant[14] and off-resonant[15] FLICFORQ, we note that, there, eduction of the static term(s) in $\mathcal{H}_{\text{QF}}$ requires precise tuning of the drive frequency *and* amplitude on each qubit. And though it is possible in the latter to adjust the gate speed, doing so involves tracing a nonlinear path in the four-dimensional space of control parameters $\omega_{1,2}^{\text{rf}}$ and $\Omega_{1,2}$. Here, a single tone is selected to match the transition frequency of the unirradiated qubit while the amplitude linearly adjusts the gate speed. Further, because the angle of the induced rotation depends on the product $\Omega_1 t$, cross resonance allows two-qubit pulses to be carefully engineered for other purposes, such as reducing bandwidth, actively combating excitations outside the computational

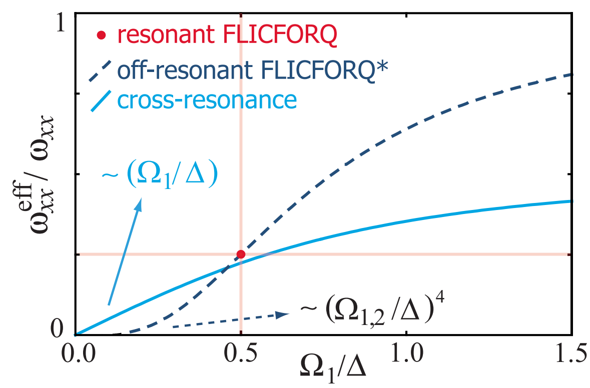

FIG. 2. (Color online) Effective interaction strength generated by cross-resonance driving as a function of drive strength-to-detuning ratio Ω_1/Δ (solid); generalized off-resonant FLICFORQ scheme of Refs. 15 and 17 (dashed); and double-resonant driving of original FLICFORQ (Ref. 14) (red point). Equal drive amplitudes ($\Omega_1=\Omega_2$) on each qubit are assumed for the latter two.

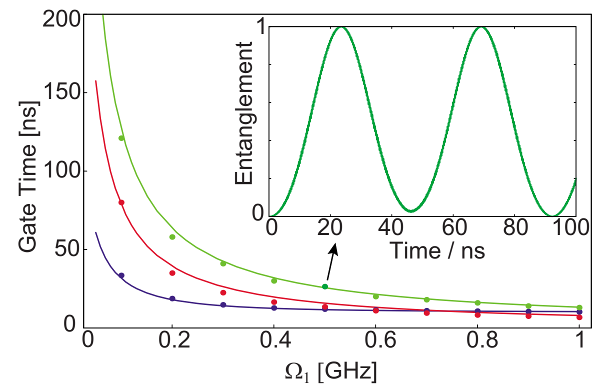

FIG. 3. (Color online) Cross-resonance two-qubit gate time (time required to generate fully entangled state from product state and vice versa) for three practical parameter sets as a function of absolute drive amplitude Ω. Green: $\{\omega_1=12.8,\ \omega_2=16.1,\ \omega_{xx}=0.13\}/2\pi$; red: $\{\omega_1=9.8,\ \omega_2=16.1,\ \omega_{xx}=0.4\}/2\pi$; and blue: $\{\omega_1=15.8,\ \omega_2=16.1,\ \omega_{xx}=0.05\}/2\pi$; all in gigahertz. Solid lines are from analytics; points are extracted from simulation of full Hamiltonian. Inset: entanglement vs time for sample point indicated by arrow at $\Omega=0.5$ GHz.

subspace,[27] or manipulating relative phase accumulations during the gate (e.g., Ref. 5). These are significant practical advantages; in essence, cross resonance gives two-qubit operations much of the flexibility of typical one-qubit gates.

We may also contrast cross resonance with the parametric pumping scheme of Bertet *et al.*[28] and the closely related version by Niskanen *et al.*[10] and Harrabi *et al.*[11] This type of coupling scheme has been experimentally tested and a linear dependence of the coupling strength on the drive amplitude in the weak driving regime has been observed.[3] This required the introduction of a nonlinear coupling subcircuit and an additional microwave port. Cross resonance also achieves tunability while keeping the qubits at optimal bias but does not require the nonlinear subcircuit nor the extra control port.

It may at first seem surprising that a dynamically tunable interaction strength is also possible for a FLICFORQ system. But the individual qubit subcircuits are themselves nonlinear. In essence, the cross-resonance scheme exploits these already present nonlinearities to achieve tunable coupling, thus circumventing the need for nonlinear coupling elements.

We have therefore found a protocol with several advantages relative to earlier microwave-controlled gate proposals. With cross resonance we need only control a single tone; the effect is fully tunable and is switched on by a frequency-only matching condition. In addition to the tunable coupling strength, cross resonance allows the coupling direction to be tuned via the microwave signal. Each of the microwave signal parameters thus plays an important role: the frequency switches on the coupling to the target qubit; the amplitude controls the gate speed; and the phase determines which two-qubit gate is implemented.

As with earlier proposals,[14,15] cross resonance can also be understood in the dressed state picture of quantum optics.[19] While the original proposal called for overlapping the upper and lower Rabi sidebands of the lower and higher frequency qubits, respectively, here cross-resonant driving ensures that the central transition at the irradiation frequency of the driven "Q1 + μ-wave photons" system matches the bare transition of the undriven Q2. We are thus creating a resonance between the central feature of the Mollow triplet[8] on Q1 and the bare transition at ω_2 of Q2. The tunability of the effective coupling strength results from the changing makeup of the Q1 + μ-wave photons hybridized eigenstates as the field amplitude is adjusted.[19]

We have compared the effective coupling strength found here to those obtained in earlier proposals involving two driving tones (see Fig. 2). When the interqubit detuning is large, absolute limits on the drive strength—imposed by the applicability of the rotating wave approximation (RWA) and the requirement to remain in the computational subspace—necessarily place us in the regime of very small Ω/Δ. There, the only previous applicable proposal, the off-resonant dual driving of Ashhab and Nori,[15] gives rise to an effective interaction that turns on only as $(\Omega/\Delta)^4$. Cross resonance produces an interaction that turns on linearly in this parameter. Working with larger detunings in turn makes it possible to use larger fixed couplings while ensuring the separability constraint $\omega_{xx}\ll\Delta$ is met, so any downward adjustment in the interaction strength from the limits on Ω may be compensated with an increased bare coupling.

The applicability of cross resonance to broadly detuned qubits bears on the fabrication process, as well. There, the challenge of overcoming the natural spread in circuit parameters is greatly reduced with the availability of a gate scheme which can accommodate a large range of detunings.

We have verified the analytic derivation of cross resonance with simulations by numerically integrating the master equation using the full laboratory-frame Hamiltonian (1). From this, we extract the two-qubit entanglement as measured by the concurrence[29] as a function of time and in turn the two-qubit gate speed under cross resonance. We find excellent agreement with the analytical results over the broad and practical range of parameters tested (see Fig. 3).

III. MICROWAVE CROSS-TALK

In superconducting circuits the signals applied to one qubit are felt also by other qubits in the register. In many im-

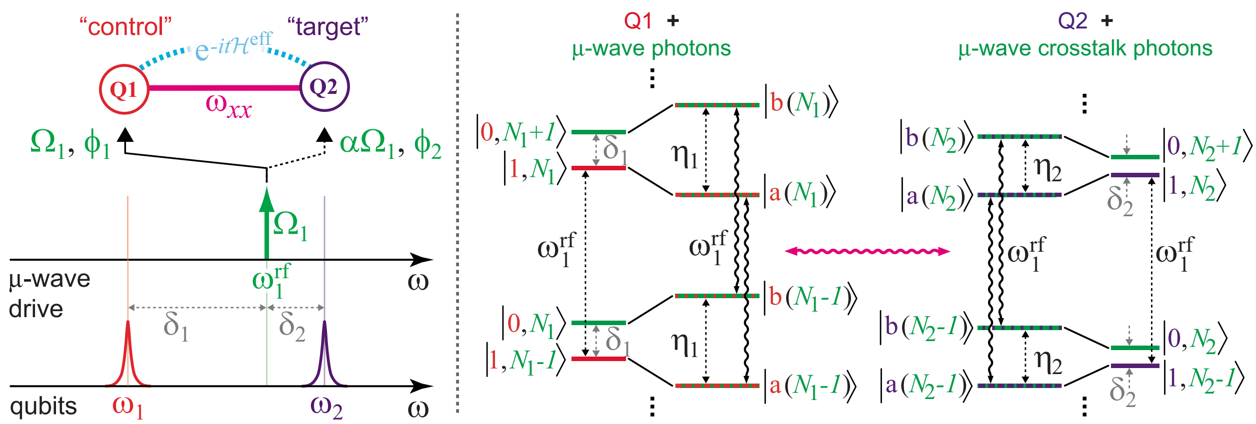

FIG. 4. (Color online) Cross resonance in presence of microwave cross-talk. Left: cross-talk leads to a fraction α of the signal applied to Q1 to be felt also by Q2. A signal at ω_1^{rf} then need not be precisely resonant with ω_2 in order to switch on the coupling. Right: dressed state manifolds for a single cross-coupled signal applied to both Q1 and Q2. As both the signal frequency and amplitude are varied, a resonance is maintained between the central transitions of each qubit+photons system.

portant implementations—most notably the circuit QED (cQED) architecture[2,30]—the basic circuit design leads to high cross-talk, as all qubits in the cavity sense the applied fields. In other cases it occurs as a result of technical difficulties in isolating nearby elements of microwave circuits.[7,31] We now show that, first, the cross-resonance effect persists in the presence of such cross-talk and second, that the cross-talk in fact relaxes the strict constraint $\omega_1^{rf}=\omega_2$ and opens up the range of frequencies $\omega_1 \lesssim \omega_1^{rf} \lesssim \omega_2$ for switching on an effective interaction.

To show the robustness, we take the case where a fraction α of the microwave signal at frequency $\omega_1^{rf}=\omega_2$ applied to Q1 is also felt by Q2. This microwave leakage signal is resonant with Q2, of amplitude $\alpha\Omega_1$, and may undergo a phase shift $\phi_2-\phi_1$ relative to the signal felt by Q1 (see Fig. 4). To describe this case we impose on $\mathcal{H}_{QF}$,

$$\{\omega_1^{rf} \to \omega_2, \quad \omega_2^{rf} \to \omega_2, \quad \xi_2 \to 0, \quad \eta_2 \to \alpha\Omega_1\}. \quad (15)$$

Then a static coupling term emerges

$$\mathcal{H}_{QF}^{eff} = \frac{1}{2}\omega_{xx}^{eff}\sigma_1^x\sigma_2^x\cos(\phi_2-\phi_1), \quad (16)$$

where ω_{xx}^{eff} is the same as in the $\alpha=0$ case of Eq. (12). Cross-talk causes the $\sigma_1^x\sigma_2^y$ term of 11 to oscillate at frequency $\alpha\Omega_1$. The above result thus relies on an RWA valid when $\alpha \gg \omega_{xx}/4\Delta$, which, given our assumption of the weak-coupling regime, corresponds to $\alpha \gtrsim 0.2$. Under weaker cross-talk the slowly oscillating term must be retained. The loss of control over the direction of the effective coupling via the microwave phase in 11 reconciles with the loss of one of our control parameters: the difference in phase of the signals felt by the qubits is no longer under the experimentalist's control, so the particular gate implemented is no longer tunable.

We now go a step further to demonstrate how cross-talk broadens this effect. The resonance established between the transitions at ω_1^{rf} of each qubit+photons system—the center features of the two Mollow triplets—does not depend on the tone being cross resonant. Because they always occur at the irradiation frequency, these transitions are resonant whenever $\omega_1^{rf}=\omega_2^{rf}$. We emphasize that this *cross-talk resonance* is in fact independent of the cross-resonant condition: whenever $\omega_1^{rf}=\omega_2^{rf}$, a time-independent term emerges in $\mathcal{H}_{QF}$, regardless of whether $\omega_1^{rf}=\omega_2$. We show this by returning again to $\mathcal{H}_{QF}$ and imposing *only* the constraint that each qubit senses the signal at frequency ω_1^{rf}, which yields the same effective Hamiltonian as shown in Eq. (16), where now the effective coupling strength involves both mixing angles,

$$\omega_{xx}^{eff} = \frac{\omega_{xx}}{2}\cos\xi_1\cos\xi_2 = \frac{\omega_{xx}}{2}\frac{1}{\sqrt{1+(\delta_2/\alpha\Omega_1)^2}\sqrt{1+(\delta_1/\Omega_1)^2}}. \quad (17)$$

This function is plotted in Fig. 5 for a range of drive frequencies and amplitudes and two cross-talk coefficients. Increasing cross-talk leads to a decreased frequency selectivity of the cross-resonance effect with respect to the microwave tone. This both broadens the range of drive frequencies capable of generating entanglement and also makes it harder to apply exclusively local gates. When $\Omega_1 \ll \Delta$, the most practically relevant range, cross-talk allows the coupling to be turned on with microwaves at *either* qubit frequency. In cQED, for example, where $\alpha \approx 1$, two qubits coupled through virtual population of the cavity can thus be entangled by irradiating the cavity at one of the qubit transition frequencies. However, the implied loss of pure local control is a very important conclusion for these large cross-talk architectures. When Ω_1 is comparable to Δ, the coupling is turned on by a tone in the vicinity of either qubit. In this case, however, other terms in $\mathcal{H}_{QF}$ become non-negligible, i.e., additional effective couplings are turned on.[32]

In superconducting qubits the standard protocol for inducing local transformations of the computational subspace is to apply resonant microwave pulses to the targeted qubit. Using resonant pulses mitigates the effects of whatever signal cross coupling is present and helps achieve the requisite individual addressability by ensuring that the signals used to manipulate the target qubit are off-resonant with the others. When the strength of the applied field is much smaller than the qubit-

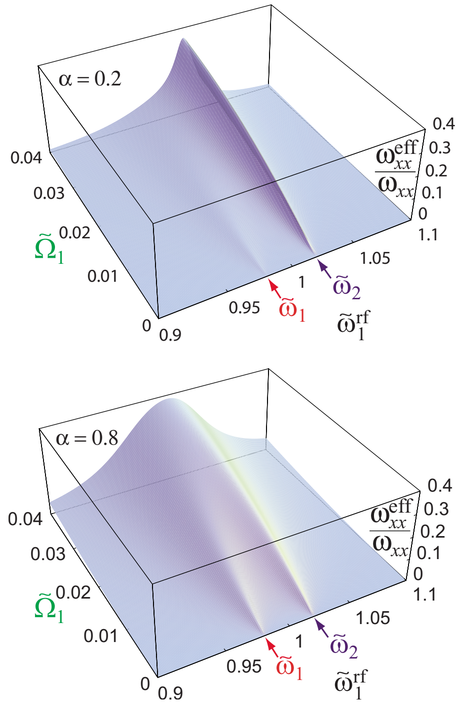

FIG. 5. (Color online) Effective cross-resonance coupling strength ω_{xx}^{eff} in presence of microwave cross-talk as a function of drive frequency $\tilde{\omega}_1^{\text{rf}}$ and drive amplitude $\tilde{\Omega}_1$ [curly bar indicating normalization to mean qubit transition frequency $\bar{\omega}=(\omega_1+\omega_2)/2$]. The microwave signal is applied intentionally to Q1 and a fraction α is felt also by Q2. Qubit transition frequencies are indicated by red ($\tilde{\omega}_1=0.98$) and purple ($\tilde{\omega}_2=1.02$) arrows. When $\alpha=0$ (not shown), the cross-resonance effect is broadened beyond finite qubit lifetime effects only by $\sqrt{N}$ fluctuations in the drive signal photon number. At top is case where $\alpha=0.2$; at bottom $\alpha=0.8$, corresponding (roughly) to the experiment of Ref. 2. In the latter case the range of drive frequencies effective at switching on the coupling is dramatically broadened. When $\alpha=1$ (not shown), ω_{xx}^{eff} is symmetric and centered at $\bar{\omega}$.

qubit detunings, a microwave pulse induces transitions only in the resonant qubit. One-qubit errors in these gates can result from an ac Stark shift-induced phase accumulation in the off-resonant qubits. Though important, these errors are well understood and may be compensated with additional dc or microwave pulses.

However, in a multiqubit register gates can be contaminated with local or nonlocal errors.[33] Because the one-qubit gates tend to be faster to implement and generally less challenging, any contamination of a two-qubit gate with a one-qubit error is not considered serious, as the error can be compensated with straightforward one-qubit rotations. But the converse is not true and contamination of a one-qubit gate with a two-qubit (i.e., nonlocal) error is a significant problem. Our results indicate that the presence of cross-talk in registers with fixed weak couplings can lead to more severe nonlocal contamination of local operations than previously expected.

Why has the effect not been more obvious? Cross resonance causes a coupling term in $\mathcal{H}_{\text{QF}}$ to become time independent and reduced in strength compared to the bare coupling. This term causes coupling effects to accumulate, rather than average out, during long irradiation pulses—the effect that makes cross resonance useful for two-qubit gates. Over the short time scales associated with single-qubit rotations of angles $\pi/2$ or smaller, the static term does not accumulate long enough to cause errors above the level $(\omega_{xx}/\Delta)^2$ present when no pulses are applied. A one-qubit rotation of angle $\theta=\Omega_1 t$ performed with a resonant microwave pulse in presence of cross-talk induces also a two-qubit rotation of angle $\omega_{xx}\alpha\theta/(2\Delta)$. Comparing this error amplitude with that present without irradiation, we find the cross-talk-induced error is larger when $\alpha\theta>1/2$. In other words, for a certain level of cross-talk there is a one-qubit rotation angle θ_c above which the cross-resonance effect causes two-qubit contamination beyond that present without pulses. If Δ and ω_{xx} are chosen to meet a certain fidelity bound, only for $\theta>\theta_c$ rotations will the effect be detrimental above that bound.

However, many widely used experimental procedures *do* rely on large-angle rotations and there cross resonance can cause anomalous results. Perhaps the most common protocol for initial characterization of a qubit system is the Rabi oscillations measurement, where a pulse of varying length is applied and the qubit state readout to accumulate statistics that can be mapped to a latitude of the single-qubit state vector on the Bloch sphere. Each cycle of the Rabi oscillation is a multiple of 2π in the rotation angle and a typical trace might have a few to many tens of oscillations. In such a procedure, for example, the cross-resonant effect will lead to qualitatively different behavior from that expected were it not accounted for. In particular, the induced entanglement will cause an amplitude modulation (AM) of the Rabi trace with an AM frequency of ω_{xx}^{eff}.

It is important to note that benchmarking,[34] a common procedure for characterizing gate fidelity,[35] would not be sensitive to this issue, as the cross-resonance effect would lead to a worse case fidelity that is unlikely to make a significant contribution to the average gate fidelity when proper randomization is implemented.[36] More generally, the cross-resonance effect must be kept in mind when using microwaves to control quantum systems, be it for one-or two-qubit gates, measurements, dynamical decoupling, or other sophisticated procedures.

Last, a note on scalability. Compared to other fixed-coupling microwave schemes, the minimal nature of cross resonance, with respect to both circuit hardware and microwave signals, makes it significantly easier to implement. However, as with other schemes for the FLICFORQ architecture, direct scaling will likely be limited to 10–20 qubits by bandwidth and finite anharmonicity of individual qubits.[14] Ultimately, the challenges of a large-scale superconducting register are likely best met by drawing on a combination of the available technologies, including cross resonance. Also, because the effect can cause two-qubit contamination of standard one-qubit gates, it is important to consider when performing even single-qubit gates in multiqubit registers.

IV. SUMMARY

We have proposed a microwave scheme for implementing two-qubit gates in superconducting qubits with fixed weak linear couplings that synthesizes some advantages of earlier two-qubit gate proposals. By irradiating the control qubit at the transition frequency of the target qubit, a resonance is established between the central transitions of the driven qubit+microwave photons system with the bare transition on the undriven qubit. We have shown that coupling to the target qubit is switched on with the drive frequency; the drive amplitude linearly tunes the effective coupling strength; and the microwave phase determines the implemented two-qubit gate. Because the angle of the induced two-qubit rotation depends on the product $\Omega_1 t$, two-qubit rotations performed with our cross-resonance scheme gain some of flexibility and ease typical of one-qubit microwave-induced rotations. We have shown that this scheme is robust in the presence of microwave cross-talk of the drive signal in the weak driving regime $\Omega \ll \Delta$ while other coupling terms begin to complicate the dynamics as Ω_1 becomes comparable to Δ. In systems with cross-talk, the cross-resonance effect can cause nonlocal contamination of one-qubit rotations implemented with resonant microwave pulses.

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